Microlocal and Global Analysis, Interactions with Geometry

10-14 February 2025, Potsdam

Organiser: Sylvie Paycha (Potsdam) with the assistance of Christian Molle

Co-organisers: Max Lein (Potsdam) Felix Medwed (Potsdam) Fabrizio Zanello (Potsdam)

Campus Golm. Picture by F. Zanello

MICROLOCAL AND GLOBAL ANALYSIS, INTERACTIONS WITH GEOMETRY

Speakers

Scientific Program

Times	Monday	Tuesday	Wednesday	Thursday	Friday
Chair	*Paycha*	*Witt*	*Lein*	*Bauer*	*Paycha*
9:00–9:50		**Schmidt**	**Jäh**	**Strohmaier**	**Schrohe**
10:00–10:50	**Bei**	**Wei**	**Witt**	**Hein**	**Hoffmann**
10:50–11:20	**Break**				
11:20–12:10	**Ronge**	**Klein**	**Dencker**	**Rosenberger**	**Seiler**
12:10–13:30	**Lunch break**				
Paycha	*Paycha*	*Seiler*		*Lein*	
13:30–14:20	**Leder**	**Penuela Diaz**		**Rouveyrol**	
14:20–14:50	**Break**				
14:50–15:40	**Kumano-go**	14:50-15:20 **Kambaso (book presentation)**		**Nakano**	
15:50–16:40	**Contini**	15:20-16:10 **Bauer**		**Han**	
16:50–17:40					
18:00			**Conference dinner**		

- **Venue:** Building 10, Campus Golm
- **Conference dinner:** Restaurant De Lewante, Gutenbergstraße 103, Potsdam

Abstracts

Wolfram Bauer

Leibniz Universität Hannover, Germany

Title: **T.B.A.**

Francesco Bei

Sapienza Universitá di Roma, Italy

Title: **L^2-harmonic forms and spinors on stable minimal hypersurfaces .**

Minimal hypersurfaces are a central topic in both Riemannian geometry and geometric analysis. We recall that a hypersurface is called minimal if it is a critical point of the volume functional and it is additionally called stable when the second variation of the volume functional is non-negative. Starting with the foundational work of Fischer and Colbrie-Schoen, the last decades have seen an intense research activity aimed to understand the geometry and topology of stable minimal hypersurfaces. In this talk I will report about some recent results concerning vanishing theorems for L^2 harmonic forms and L^2 harmonic spinors on stable minimal hypersurfaces. The talk is based on a joint work with Giuseppe Pipoli.

Alessandro Contini

Leibniz Universität Hannover, Germany

Title: **SG-pseudo-differential operators and symplectic structures.**

In this talk I will explore some aspects of the calculus of SG-pseudo-differential operators, in particular I will show how it relates to a symplectic structure on a manifold with corners. Time permitting, I will show how one can construct some explicit examples and discuss whether this is always possible.

Nils Dencker

Lund University, Sweden

Title: **The generic instability of differential operators.**

It came as a surprise when Hans Lewy in 1957 presented a non-vanishing smooth complex vector field on $\mathbb{R}^3$ that is not solvable anywhere. After all, the classical Cauchy-Kowalevskaya Theorem shows that any analytic PDE is solvable in the analytic category. Actually, the Lewy vector field is the tangential Cauchy-Riemann operator on the boundary of a strictly pseudoconvex domain in $\mathbb{C}^2$. Hörmander showed in 1960 that almost all linear PDEs are not solvable by proving that the non-generic vanishing of the Poisson bracket is necessary for solvability. This bracket condition for PDEs has many consequences for the kernel, the range and the spectral stability of linear PDEs and the stability of the quasilinear Cauchy problem. A fifty year development lead to the proof of the Nirenberg-Treves conjecture: that principal type differential operators are solvable if and only if condition (Ψ) on the principal symbol is satisfied. This is a condition on the sign changes of the imaginary part of the principal symbol along the bicharacteristics of the real part. For non-principal type differential operators, a condition similar to (Ψ) on the refined principal symbol (which includes the subprincipal symbol) is necessary for solvability. Recently, it has been proved this condition is also sufficient for solvability of non-principal type differential operators having real principal symbols vanishing of second order.

Zhicheng Han

Leibniz Universität Hannover, Germany

Title: **On the Laplacian and Dirac spectrum of certain Lie groups, and what they are good for.**

In this talk I will give some computational results on the spectra of differential form Laplacian and Dirac operators on some Lie groups such as nilpotent groups as well as semisimple Lie groups. In some cases, we can deploy representation-theoretic tools to obtain more spectral information. If the time permits I will show to the authors in cases such as H-type groups how we can hope to compute the spectral asymptotics and as a consequence derived some results of Rumin using purely analytic approaches.

TBA.

Hans-Joachim Hein

Universität Münster, Germany

Title: **A gluing construction for complex surfaces with hyperbolic cusps .**

We will describe an example of a degeneration of degree 6 algebraic surfaces in with an isolated triple point singularity on its central fiber. Then we will show how the unique negative Kähler-Einstein metrics on the smooth fibers, which exist by the Aubin-Yau theorem, disintegrate into three distinct geometric pieces on approach to the central fiber:

(1) Kobayashi's complete Kähler-Einstein metric on the complement of the triple point,
(2) long thin neck regions, and
(3) Tian-Yau's complete Ricci-flat Kähler metric in small neighborhoods of the vanishing cycles.

Joint work with Xin Fu and Xumin Jiang.

Arne Hoffmann

Leibniz Universität Hannover, Germany

TITLE: **Relative Trace Formulas in Geometry and Physics.**

Wave trace invariants are spectral invariants that can be used to study both compact and noncompact manifolds. In the case of scattering by a finite number of compact obstacles, it is useful to consider a variant, the relative wave trace. Its leading singularities are related to the asymptotic decay of the function Xi, which has been studied by (Fang-Strohmaier, 2021) and (Hanisch-Strohmaier-Waters, 2022). The function Xi also appears as the energy term which causes the Casimir effect in physics. In this talk, I will introduce the function Xi, its connections to geometry and physics, and remark on recent progress concerning non-Dirichlet boundary conditions, especially transmission boundary conditions.

Christian Jäh

Georg-August-Universität Göttingen, Germany

TITLE: **Microlocal analysis around radial points.**

We report on progress in an ongoing project (joint work with Ingo Witt) about operators of non-principal type. In particular parametrices around radial points and questions of propagation of singularities.

Christian Klein

Institut de Mathématiques de Bourgogne, France

TITLE: **Polarization approach to certain singular integrals.**

The inverse scattering approach to the integrable Davey-Stewartson equations and the reconstruction of the conductivity in electrical impedance tomography require the solution of a Dirac system with highly oscillatory factors. The latter can be solved via a Neumann series involving singular integrals. Standard steepest descent techniques cannot be applied when these singularities are close to the stationary points. In these cases we propose a polarization approach to obtain an asymptotic solution . This is work with J. Sjöstrand and M. Zerzeri.

Naoto Kumano-go

Kogakuin University of Technology and Engineering, Japan

TITLE: **Phase space Feynman path integrals of parabolic type on the torus as analysis on path space.**

We provide two general sets of functionals for parabolic phase space Feynman path integrals on the torus, which have a mathematically rigorous and well-defined interpretation. More specifically, for each functional in each set, the time slicing approximation of its phase space path integral converges uniformly on compact subsets of the product space defined by the final points of position paths and the initial points of momentum paths. The two sets of functionals are closed under addition, multiplication, translation of paths, invertible integer linear transformations, and functional differentiation. As a result, it is possible to create a large number of path integrable functionals. Furthermore, while caution is necessary when applying these properties, we ensure that the following operations are valid for the phase space path integrals: (1) interchange of the order of phase space path integrals with integration with respect to time, (2) interchange of the order of phase space path integrals with limits, (3) invariance under orthogonal integer transformations of paths, (4) invariance under translations with respect to momentum paths, and (5) integration by parts in the context of functional differentiation with respect to position paths.

Roee Leder

The Hebrew University of Jerusalem, Israel

Title: **Hodge-like theory for geometric overdetermined boundary value problems.**

In this talk, I will introduce the theory of elliptic pre-complexes, which generalizes several aspects of the classical theory of elliptic complexes. In essence, elliptic pre-complexes are sequences of pseudodifferential boundary value problems of varying orders, interacting through generalized Green's formulas. They involve the notion of overdetermined ellipticity and, most prominently, a geometric condition emergent from curvature, which we call the "order reduction property". This property replaces the classical requirement that the sequence forms a cochain complex, allowing Hodge-like theories to manifest for a broader class of geometric overdetermined problems, regardless of the background Riemannian structure. Applications include cohomological formulations for the uniqueness and solvability of various boundary value problems involving exterior covariant derivatives, as well as for the linearized forms of the prescribed Riemannian curvature and Einstein tensor problems. This linear analysis provides a foundation for addressing the corresponding nonlinear problems.

Alejandro Penuela Diaz

Universität Potsdam, Germany

Title: **Rigidity and Monotonicity of the Hawking Energy on Hawking Surfaces.**

The Hawking energy is one of the simplest quasi-local energy definitions in general relativity. Despite its simplicity, the Hawking energy has faced challenges due to ambiguities when applied to general surfaces. In this talk, I will present my recent results demonstrating that the Hawking energy exhibits key physical and mathematical properties—non-negativity, rigidity, and monotonicity—when evaluated on Hawking surfaces, a class of critical surfaces of the Hawking functional. These results establish Hawking surfaces as useful tool for evaluating the Hawking energy and reinforce its potential as a meaningful tool for understanding gravitational phenomena.

Marvin Schmidt

Georg-August-Universität Göttingen, Germany

Title: **Global pseudodifferential calculus on the 6-dimensional nilpotent Lie group $G_{6,16}$.**

Fischer and Ruzhansky [1] discussed a global quantization on graded Lie groups G such that one gets an operator calculus, in the sense that the operators form an algebra, stable under taking adjoints and acting on an adapted scale of Sobolev spaces. Moreover, the operators act continuously on the Schwartz space. In this calculus, the amplitude functions $\sigma = \sigma(g, \pi)$, $(g, \pi) \in G \times \operatorname{supp}\mu$, where $\operatorname{supp}\mu \subseteq \widehat{G}$ is the support of the Plancherel measure, are operator-valued and satisfy certain estimates involving a positive Rockland operator. On the Heisenberg group $\mathbb{H}^d$, one can rewrite those amplitude functions via the Weyl quantizaton on $\mathbb{R}^d$ as $\sigma(g, \pi) = \mathrm{Op}^W(a_{g,\pi})$, and transfer all properties of σ to properties of the smooth complex-valued function $a = a_{g,\pi}(u, \xi)$ on $\mathbb{H}^d \times \operatorname{supp}\mu \times \mathbb{R}^d \times \mathbb{R}^d$, where now $\operatorname{supp}\mu \cong \mathbb{R} \setminus \{0\}$. Potentially, this approach works for all nilpotent Lie groups having flat coadjoint orbits.

In this talk, we construct such a calculus on $G_{6,16}$ (see Nielsen's list [3] of low-dimensional nilpotent Lie groups) using the magnetic Weyl quantization [2] with a constant magnetic field.

[1] V. Fischer and M. Ruzhansky. *Quantization on nilpotent Lie groups.* Vol. 314. Progress in Mathematics. Birkhäuser/Springer, [Cham], 2016, pp. xiii+557.
[2] M. Măntoiu and R. Purice. "The magnetic Weyl calculus". In: *J. Math. Phys.* 45.4 (2004), pp. 1394–1417.

[3] O. A. Nielsen. *Unitary representations and coadjoint orbits of low-dimensional nilpotent Lie groups.* Vol. 63. Queen's Papers in Pure and Applied Mathematics. Queen's University, Kingston, ON, 1983, pp. xiii+117.

Elmar Schrohe

Leibniz Universität Hannover, Germany

Title: **The plasmonic eigenvalue problem and the Dirichlet-to-Neumann operator on manifolds with fibered cusps.**

A plasmon of a bounded domain $\Omega \subseteq \mathbb{R}^n$ is a non-trivial harmonic function on $\mathbb{R}^n \setminus \partial\Omega$ which is continuous at $\partial\Omega$ and whose interior and exterior normal derivative at $\partial\Omega$ have a constant ratio. This ratio is called a plasmonic eigenvalue of Ω. It is indeed an eigenvalue of $N_+^{-1}N_-$, where $N_\pm$ denote the exterior and interior Dirichlet-to-Neumann operators.
Our long term goal is to understand this problem on a manifold with fibered cusp singularities. A prototypical example would be the complement of two touching strictly convex domains in $\mathbb{R}^n$. Clearly, the problem requires a precise analysis of the Dirichlet-to-Neumann operator in this setting. In a first step, we consider the Calderòn projector for general elliptic differential operators of arbitrary order associated with this type of singularity, so-called ϕ-differential operators. We show that the Calderòn projector is a ϕ-pseudodifferential operator in the sense of Mazzeo and Melrose. Next we study the Dirichlet-to-Neumann operator for Laplacians associated with fibered cusp metrics and obtain that it also is a ϕ-pseudodifferential operator of order one.
This is a report on ongoing work with Karsten Fritzsch and Daniel Grieser.

Joerg Seiler

Università degli Studi di Torino, Italy

Title: **Calculus for parametric boundary problems with global projection conditions.**

A pseudodifferential calculus for parameter-dependent operators on smooth manifolds with boundary in the spirit of Boutet de Monvel's algebra is constructed. The calculus contains, in particular, the resolvents of realizations of differential operators subject to global projection boundary conditions (spectral boundary conditions are a particular example); resolvent trace asymptotics are easily derived. The calculus is related to but different from the calculi developed by Grubb and Grubb-Seeley. We use ideas from the theory of pseudodifferential operators on manifolds with edges due to Schulze, in particular the concept of operator-valued symbols twisted by a group-action. Parameter-ellipticity in the calculus is characterized by the invertibility of three principal symbols: the homogeneous principal symbol, the principal boundary symbol, and the so-called principal limit symbol. The principal boundary symbol has, in general, a singularity in the co-variable/parameter space, the principal limit symbol is a new ingredient of our calculus.

Alexander Strohmaier

Leibniz Universität Hannover, Germany

Title: **Locality in quantum field theory on curved spacetimes and microlocal analysis.**

The local structure of quantum fields in curved spacetimes is intimately related to unique continuation theorems in analytic microlocal analysis. I will review the mathematical results on unique continuation and then explain how they relate to quantum field theory on analytic curved spacetimes. (Based on joint work with E. Witten)

Yawei Wei

Nankai University, China

Title: **Existence results for cone degenerate p-Laplace equation.**

In this talk, we study the cone degenerate p-Laplace equation, which is based on the analysis of the manifold with conical singularities from Prof. Schulze. Here we provide the existence of viscosity solutions for Dirichlet problem of cone degenerate p-Laplace equation by proving Harnack inequality, Alexandrov-Bakelman-Pucci estimates and Hölder estimates. Furthermore, we obtain the existence of distribution solutions by proving the bounded viscosity solution is also a distribution solution.

Ingo Witt

Georg-August-Universität Göttingen, Germany

Title: **Local well-posedness for the 3D compressible resistive equations of magnetohydrodynamics.**

The compressible magnetohydrodynamics (MHD) equations model electrically conducting fluids. They constitute a nonlinear system of equations for $(\rho, \mathbf{v}, \mathbf{B}, p)$, where ρ is mass density, $\mathbf{v}$ is fluid velocity, $\mathbf{B}$ is the magnetic field, and p is pressure. The simplest model are the ideal MHD equations, where one assumes an infinite electric conductivity. This model is widely studied in the mathematical literature. Alfvén's frozen-in flux theorem holds in that case which states that the fluid and the magnetic field are tied to each other in a way that the topology of the magnetic field cannot change over time. From a physical point of view, the conditions for the ideal MHD approximation break down at current sheets. The resistive MHD equations having finite electric conductivity and thus introducing magnetic diffusivity $\eta > 0$ to the model, no matter how small, are then still a good approximation. In particular, they allow for magnetic reconnection, see [3]. Magnetic reconnection is responsible for the release of stored magnetic energy as can be observed, for instances, in solar flares.

Mathematically, the resistive MHD equations form a hyperbolic-parabolic system (as opposed to the purely hyperbolic ideal MHD equations). Specifically, the magnetic field $\mathbf{B}$ fulfills the induction equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \, \Delta \mathbf{B}$$

along with the constraint $\nabla \cdot \mathbf{B} = 0$. The standard theory on hyperbolic-parabolic systems, see [2], does not apply in this case.

There are six basic geometric arrangements of increasing mathematical complexity, listed in [1, Section 4.6], where one would like to have some mathematical theory. We treat the first of these arrangements, that of a perfectly conducting wall, and prove local in time well-posedness. The corresponding results for the ideal MHD equations were obtained in [4].

Joint work with Rayhana Darwich (Lebanese Association for Scientific Research (LASeR), Tripoli, Lebanon).

[1] J. P. Goedbloed and S. Poedts, *Principles of Magnetohydrodynamics. With Applications to Laboratory and Astrophysical Plasmas.* Cambridge Univ. Press, Cambridge, 2004.

[2] S. Kawashima, *Systems of a hyperbolic-parabolic composite type, with applications to the equations of magnetohydrodynamics*. Ph.D. thesis, Kyoto Univ., Kyoto, 1984.

[3] E. Priest and T. Forbes, *Magnetic Reconnection: MHD Theory and Applications*. Cambridge Univ. Press, Cambridge, 2000.

[4] T. Yanagisawa and A. Matsumura, *The fixed boundary value problems for the equations of ideal magnetohydrodynamics with a perfectly conducting wall condition*. Comm. Math. Phys. **136** (1991), 119–140.